## УПРАВЛІННЯ ПІДПРИЄМСТВОМ

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# MODEL OF DETERMINING THE OPTIMAL SUPPLY TIME OF PRODUCTS

***Urgency of the research.*** *Considerable amounts of money are spent on maintaining inventory at enterprises, so it is strategically important to manage them effectively. One of the methods lies in defining the optimal moment of supplying new shipments.*

***Target setting.*** *Optimizing the new shipment supply moment is a problem that was considered under deterministic conditions. However, in reality a set of random factors have a significant influence, so it is necessary to take them into account.*

***Actual scientific researches and issues analysis.*** *An important contribution to the modelling of supply processes was made by R. Wilson, J. Headley, J. Shapiro, W. Baumol, J. Tobin, M. Miller, D. Orr and others.*

***Uninvestigated parts of general matters defining.*** *The existing models do not take into account the stochastic nature of demand.*

***The research objective.*** *The main goal is to present a new probabilistic model of supplying goods and to optimize the moment of the new shipment under conditions of stochastic demand.*

***The statement of basic materials.*** *We minimize the loss function which takes into account both cases: when the new shipment was delivered both before and after the actual running out of products (i.e. the storage expenses and deficit losses). Under conditions of normality of the actual moment of running out of goods we obtain the explicit form for the optimal supply moment.*

***Conclusions.*** *The stochastic model for the new shipment supply moment was presented. The optimal moment of supply was found under the condition of normality of the moment of running out of goods.*

***Keywords:*** *stochastic models; demand; inventory control; profit; optimization.*

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# МОДЕЛЬ ВИЗНАЧЕННЯ ОПТИМАЛЬНОГО МОМЕНТУ ПОСТАВКИ ПРОДУКЦІЇ

***Актуальність теми дослідження.*** *Підприємства витрачають значні суми на утримання запасів, тому ефективне управління ними має стратегічну важливість. Один із методів полягає у визначенні моменту поставки товарів.*

***Постановка проблеми.*** *Задача оптимізації моменту поставки нової партії розглядалася в детермінованих умовах. Однак, в реальності значний вплив має набір випадкових факторів, тому його необхідно враховувати.*

***Аналіз останніх досліджень і публікацій.*** *Важливий внесок у моделювання процесів поставки зробили Р. Вілсон, Дж. Хідлі, Дж. Шапіро, В. Баумоль, Дж. Тобін, Д. Орр та інші.*

***Виділення недосліджених частин загальної проблеми.*** *Існуючі моделі не враховують стохастичну природу попиту.*

***Постановка завдання.*** *Головною метою є введення нової імовірнісної моделі поставок продукції та оптимізація моменту поставки нової партії в умовах стохастичного попиту.*

***Викладення основного матеріалу.*** *Ми мінімізуємо функцію витрат, яка враховує обидва випадки: поставок нової партії як раніше фактичного закінчення товару, так і пізніше за це (тобто витрат на зберігання та втрат дефіциту). За умови нормальності справжнього моменту закінчення товару ми отримали явний вигляд оптимального моменту поставки.*

***Висновки.*** *Представлена нова модель поставки продукції. Було знайдено оптимальний момент поставки за умови нормальності моменту закінчення товару на складі.*

***Ключові слова:*** *стохастична модель; попит; управління запасами; прибуток; оптимізація.*

**Urgency of the research.** Due to the considerable amounts spent on maintaining inventory at the enterprises, the problem of their effective management is strategically important. One of the methods lies in defining the optimal moment of supplying new shipments. The best known models for now are the Baumol-Tobin and Miller-Orr models, but the solution of this class of problems using these models is associated with difficulties of calculation nature. Therefore, the development of the new inventory management models is a relevant and significant problem in a context of intense competition in consumer markets.

**Target setting.** The main problem of the inventory management is the rationalization, that is, finding the optimal balance between losses from fund freezing and the level of customer satisfaction. To solve this problem, a set of economic and mathematical models has been developed. One of the distinctive features of creating an adequate inventory management model is the fact that in real life we often encounter probabilistic factors that affect the logistics system as a whole. Consequently, in the context of a particular economic system, there is a need to find an optimal solution in conditions of all kinds of uncertainty.

**Actual scientific researches and issues analysis.** At present, a number of works are devoted to the correct formulation and management of inventory, among which, first and foremost, are the works of R. Wilson, J. Headley, J. Shapiro and others. Modern approaches to this problem are presented in [1-5]. As for stochastic inventory management, the papers [6-8] should be mentioned. According to them, today analytical methods and methods of simulation modeling are used in solving such problems, while the accuracy and completeness of the solution depends on the initial conditions of the original problem and on the correctness of its formulation.

**Uninvestigated parts of general matters defining.** It should be noted that the previous researches presented mostly deterministic inventory management models. However, in real conditions there is always a probabilistic component, which makes the considered process random. From the mathematical point of view, the problem of taking uncertainty into account in optimization problems is rather complex, which is explained by the use of numerical methods. And in all publications above, authors suggest to solve such problems using numerical methods, leaving behind the fact that their use in practice is quite problematic. Therefore, from the practical point of view, the creation of generalizing models that take into account uncertain factors and allow for an analytical solution is of particular interest.

**The research objective.** The main goal of this paper is to present a new probabilistic model of supplying goods and to optimize the moment of the new shipment under conditions of stochastic demand. The correct formulation and solution of the problem under uncertainty helps to reduce the amount of stocks and allows for the right management decisions.

**The statement of basic materials.** Consider a stochastic model for determining the volume of supply of some product, taking into account the uncertainty of demand for this product. We will assume that the probability of delay and untimely supply of goods tends to zero (the goods are supplied at the order time $\tau$). The time when the supplier runs out of the goods is calculated by the formula

$$\gamma = \gamma_0 + \Delta\gamma, \quad (1)$$

where $\gamma_0$ is expected time of running out, $\Delta\gamma$ is a random deviation from the expected time.

Next, we assume that the random variable $\Delta\gamma$ has a normal distribution with parameters ($M = 0, \sigma > 0$). With this in mind, $\gamma$ is also a normally distributed random variable with parameters ($M = \gamma_0, \sigma$).

In the model presented, the cost function includes the expenses of storing and deficit losses, which characterize the lost profit. Assume that they are proportional to the time of absence of the required quantity of goods in the warehouse. The cost of storing goods in volume $V$ after supplying in the interval of time until the moment $\gamma$ of real running out of the product in the case when the supply occurred at an earlier moment $\tau^*$ ($\tau^* < \gamma$) is

$$P = \beta V(\gamma - \tau^*), \quad (2)$$

where $\beta = const$ is the daily storage cost of a unit of product.

In the case of incomplete demand satisfaction $\tau^* > \gamma$, deficit losses of the product arise during the time interval from the real moment $\gamma$ of running out of the product to the moment $\tau^*$ of the volume $V$ supply:

$$K = \frac{V}{\gamma_0}\mu(\tau^* - \gamma), \quad (3)$$

where $\mu = const$ is profit from the sale of a unit of the supplied product, $\frac{V}{\gamma_0}$ characterizes the average daily volume of goods sold.

The resulting total expenses can be calculated by the formula

$$P + K = \begin{cases} V\beta(\gamma - \tau^*),\ if\ \gamma > \tau^*, \\ \frac{V}{\gamma_0}\mu(\tau^* - \gamma),\ if\ \tau^* > \gamma. \end{cases} \quad (4)$$

Due to the fact that in stochastic models the function of total costs is random, we will consider its mathematical expectation as its characterization. In the presented model with a continuous random variable $\Delta\gamma$, which characterizes the demand uncertainty and has a normal distribution, the mathematical expectation of the total expenses has the form

$$Z(\tau^*) = \int_{-\infty}^{\tau^*-\gamma_0} \frac{V}{\gamma_0}\mu(\tau^* - \gamma_0 - \Delta\gamma)\theta(\Delta\gamma)d\Delta\gamma + \int_{\tau^*-\gamma_0}^{+\infty} \beta V(\gamma_0 + \Delta\gamma - \tau^*)\theta(\Delta\gamma)d(\Delta\gamma) \quad (5)$$

The presented problem (5) reduces to the search of the supply moment $\tau^*$, in which the mathematical expectation of the total expenses acquires the minimum value.

Consider the first summand of (5):

$$Z_1(\tau^*) = \frac{V}{\gamma_0}\mu\tau^*\Phi\left(\frac{\tau^*-\gamma_0}{\sigma}\right) - V\mu\Phi\left(\frac{\tau^*-\gamma_0}{\sigma}\right) - \frac{V}{\gamma_0}\mu\int_{-\infty}^{\tau^*-\gamma_0} \Delta\gamma\theta(\Delta\gamma)d(\Delta\gamma), \quad (6)$$

where $\Phi(x)$ is a well-known Laplace function.

After transforming the integral (6), we obtain:

$$Z_1(\tau^*) = \frac{V}{\gamma_0}\mu\tau^*\Phi\left(\frac{\tau^*-\gamma_0}{\sigma}\right) - V\mu\Phi\left(\frac{\tau^*-\gamma_0}{\sigma}\right) + \frac{V}{\gamma_0}\mu\frac{1}{\sqrt{2\pi}}e^{-\frac{(\tau^*-\gamma_0)^2}{2\sigma^2}} \quad (7)$$

Similarly, the second summand of (5) has the following form:

$$Z_2(\tau^*) = \beta V\gamma_0\left(1 - \Phi\left(\frac{\tau^*-\gamma_0}{\sigma}\right)\right) + \beta V\frac{1}{\sqrt{2\pi}}e^{-\frac{(\tau^*-\gamma_0)^2}{2\sigma^2}} - \beta V\tau^*\left(1 - \Phi\left(\frac{\tau^*-\gamma_0}{\sigma}\right)\right) \quad (8)$$

Taking into account (7) and (8), we can rewrite (5):

$$Z(\tau^*) = Z_1(\tau^*) + Z_2(\tau^*) =$$

$$= \frac{V}{\gamma_0}\mu\tau^*\Phi\left(\frac{\tau^* - \gamma_0}{\sigma}\right) - V\mu\Phi\left(\frac{\tau^* - \gamma_0}{\sigma}\right) + \frac{V}{\gamma_0}\mu\frac{1}{\sqrt{2\pi}}e^{-\frac{(\tau^*-\gamma_0)^2}{2\sigma^2}} +$$

$$+\beta V\gamma_0\left(1 - \Phi\left(\frac{\tau^*-\gamma_0}{\sigma}\right)\right) + \beta V\frac{1}{\sqrt{2\pi}}e^{-\frac{(\tau^*-\gamma_0)^2}{2\sigma^2}} - \beta V\tau^*\left(1 - \Phi\left(\frac{\tau^*-\gamma_0}{\sigma}\right)\right) \quad (9)$$

To find the minimal value of expenses, let us find the roots of the derivative of (9):

$$\tau^* = \gamma_0 + \sigma\Phi^{-1}\left(\frac{\gamma_0\beta}{\mu+\gamma_0\beta}\right), \quad (10)$$

where $\gamma_0$ is the expected moment of running out of goods, $\beta$ is a unit storage cost, $\mu$ is profit from the sale of a unit of the supplied product.

From (10) it follows that the optimal moment of supply of a new shipment is shifted relatively to the time point $\gamma_0$ by a value which depends on $\mu$, $\beta$ and parameters of the normal distribution of $\Delta\gamma$, that characterizes the deviation from the expected time of running out of goods.

If $\beta = 0.5$, the expected time of running out of goods $\gamma_0 = 10$, and $\Delta\gamma \sim \mathcal{N}(0{,}3)$, the optimal moment of supply $\tau^*$ is calculated for the range of values $\mu$ of profit from the sale of a unit of the supplied product in Tab.1.

*Table 1*

**The calculation of the optimal supply time $\tau^*$**

| $\mu$ | $\frac{\gamma_{0\beta}}{\mu+\gamma_0\beta}$ | $\sigma\Phi^{-1}\left(\frac{\gamma_{0\beta}}{\mu+\gamma_0\beta}\right)$ | $\tau^*$ | $\mu$ | $\frac{\gamma_{0\beta}}{\mu+\gamma_0\beta}$ | $\sigma\Phi^{-1}\left(\frac{\gamma_{0\beta}}{\mu+\gamma_0\beta}\right)$ | $\tau^*$ |
|---|---|---|---|---|---|---|---|
| 10 | 0,833 | 3,87 | 13,87 | 60 | 0,454 | -0,46 | 9,54 |
| 20 | 0,714 | 2,26 | 12,26 | 70 | 0,416 | -0,84 | 9,16 |
| 30 | 0,625 | 1,27 | 11,27 | 80 | 0,384 | -1,17 | 8,83 |
| 40 | 0,555 | 0,56 | 10,56 | 90 | 0,357 | -1,46 | 8,54 |
| 50 | 0,500 | 0,00 | 10,00 | 100 | 0,333 | -1,72 | 8,28 |

As we can see, the bigger realization profit is, the earlier the new shipment has to be. Also note that in our example, the value $\mu = 50$ is somewhat "the point of stability" meaning that $\tau^* = \gamma_0$.

**Conclusions**. The presented model allows to obtain an analytical expression, which determines the day of delivery of a new shipment in conditions of random demand. The numerical results of the research are presented. Application of the received model will minimize the expected costs. The obtained analytical expression, in contrast to cumbersome numerical calculations, allows us to solve a number of optimization problems in practice.

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