ЕКОНОМІКА ТА УПРАВЛІННЯ НАЦІОНАЛЬНИМ ГОСПОДАРСТВОМ

UDC 369.04:519.863

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ASSESSMENT OF THE INSURANCE COMPANY WITH LAPLACE TRANSFORM

Abstract. *A model of assessment of the solvency of the insurance company is designed in the paper. For a wide class of tasks it is shown that obtaining an analytical expression for the function of payment magnitude distribution is possible only with the use of numerical methods, including algorithm due to Dufresne and Gerber. It is indicated that if the value of payments has an exponential distribution, there is a possibility of obtaining an analytic solution of the problem by means of inverse Laplace transforms. The probability of stable operation of the insurance company is discovered, as well as a graphical representation of the results depending on the distribution parameters is achieved.*

Keywords: *insurance; the exponential distribution; probability; Laplace transform.*

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ОЦІНКА РОБОТИ СТРАХОВОЇ КОМПАНІЇ ЗА ДОПОМОГОЮ ПЕРЕТВОРЕННЯ ЛАПЛАСА

Анотація. *Побудовано модель оцінки платоспроможності страхової компанії. Для широкого класу задач показано, що отримання аналітичного виразу для функції розподілу величини виплат можливе тільки з використанням чисельних методів, включаючи алгоритми Дюфресне та Гербера. Показано, що у випадку, коли величина виплат має експоненційний розподіл, можливе отримання аналітичного розв'язку поставленої задачі за допомогою перетворення Лапласа. Знайдено ймовірність стабільної роботи страхової компанії, а також отримано графічне представлення результатів у залежності від параметрів розподілу.*

Ключові слова: *страхування; експоненційний розподіл; імовірність; перетворення Лапласа.*

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ОЦЕНКА РАБОТЫ СТРАХОВОЙ КОМПАНИИ ПРИ ПОМОЩИ ОБРАТНОГО ПРЕОБРАЗОВАНИЯ ЛАПЛАСА

Аннотация. *Построена модель оценки платежеспособности страховой компании. Для широкого класса задач показано, что получение аналитического выражения для функции распределения величины выплат возможно только с использованием численных методов, включая алгоритмы Дюфресне и Гербера. Показано, что в случае, если величина выплат имеет экспоненциальное распределение, возможно получение аналитического решения поставленной задачи при помощи обратных преобразований Лапласа. Найдена вероятность стабильной работы страховой компании, а также получено графическое представление результатов в зависимости от параметров распределения.*

Ключевые слова: *страхование; экспоненциальное распределение; вероятность; преобразование Лапласа.*

Urgency of the research. Financial stability of the insurance sector is an important condition for a stable macro-economic development and welfare of society as a whole.

The ability of the insurance company to meet its obligations under various adverse changes in market conditions, that is, to be financially sustainable is important to maintain the efficiency of the entire chain of economic relations in the society. On account of the problem of financial stability of insurance companies on the Ukrainian market has become particularly relevant today. The perfection of methods for evaluating the risks of the insurance company and the correct formation of the insurance premium largely determines its level of competitiveness. Thus, the justification of administrative decisions in the insurance companies plays special role simulation models, which are based on probabilistic and statistical methods. At the same time as a measure of insurance risk the probability of bankruptcy of the company is accepted. The study of the insurance activities on probabilistic and statistical models allows performing the calculation of indicators such as the size of the insurance premium, bankruptcy probability under various scenarios of occurrence of insurance cases, the value of the insurance reserve, which ultimately enables developing the optimal management of the insurance company.

Target setting. When considering the minimization of risks in insurance it is necessary to develop an efficient and simple probabilistic assessment method based on the achieving the analytical solution of the problem.

Actual scientific researches and issues analysis. Questions regarding the search of new approaches to the maintenance of financial stability of insurance companies and risk management has been covered in the scientific papers of recent years. It should be noted that at the moment there are two classic risk models of insurance companies: the model of individual risk (static) and the collective risk model (dynamic). As shown by A. N. Shiriaiev [1], in the framework of models of the first type the problem solving is limited to the calculation of the probability of exceeding the level of the sum of all payments for all claims of the insurance portfolio. The author suggests a mechanism for the iterative calculation for the bankruptcy probability for the model of Ya Gambosh. In contrast to the first type of models, the second model types are not comprehensively explored. The most interesting results in the framework of dynamic models are obtained by the authors from following references: [2; 3; 4], and the authors from [5; 6] use different methods of asymptotic theory, stochastic processes and queuing theory to solve the set tasks.

Uninvestigated parts of general matters defining. Obtaining the analytical solutions of dynamic problems and justification of its use for the evaluation of the insurance company's risk is complicated, from a mathematical point of view, and the correct formulation of which is quite difficult. For this reason, according to the actual scientific researches and issues, in modern actuarial mathematics there is no universal method for its determination.

The research objective. The objective of the paper is to develop an effective and simple method to produce a probabilistic assessment of the risk of the insurance company based on a study of the analytical model using mathematical tools of complex analysis.

The statement of basic materials. It is known that one of the basic mathematical models of risk theory is a classical insurance model of Lundberg - Cramer [1]. This model enables to analyze the bankruptcy and to determine the probability of the insurance company performing its obligations under insurance contracts in time, that is, at the moment of insurance cases where the *u* is an initial capital, with - the intensity of contributions, θ_k - sequence of independent identically distributed random variables that characterize the size of payments to the mathematical expectation m and the distribution function

Let the number of payments over a time interval (0, t) is a Poisson process

$$N_t = \sum_k I(T_k \le t), \tag{2}$$

where $I\,(T_k \le t)$ the number of claims that are made of the insurance company in times T_k $I\,(T_k \le t)$ can take the values 0 or 1. The times of receipt of requirements for payment $(T_{k+1} - T_k)_{k\ge1}$ are independent random variables and have an exponential distribution law with parameter λ

$$P(T_{k+1} - T_k \le t) = \begin{cases} 1 - e^{-\lambda t}, & t \ge 0 \\ 0, & t < 0 \end{cases} \tag{3}$$

By designating R_t a capital of the insurance company at time t, we obtain

$$MR_t = u + (c - \lambda m)t$$

Taking into account the load factor $\gamma = \frac{c}{\lambda m} - 1$ expression for the rate of premium income will be as follows:

$$c = (1+\gamma)\lambda m$$

Considering the total probability formula and properties of ordinary Poisson process for the function (4), we obtain the corresponding integral-differential equation

$$\lambda\Phi(\omega) = c\Phi'(\omega)\lambda + \int_0^\infty \Phi(\omega - x) f(x)dx, \quad (5)$$

where $f(x) = F'(x)$ $corresponds\ to\ the$ the density distribution of the value of payments. A number of authors [2, 3] state that if the payout values possess the arbitrary distribution law it is difficult to obtain an analytical expression for $\Phi(\omega)$. In this case, the solution is found numerically using Dufresne algorithm [1]. We show further that for some types of distribution F (ω) still it is quite possible to obtain an analytical solution. Employing the Laplace transform with the equation (5), we obtain

$$\lambda\Phi(\omega) = c\big(p\psi(p) - \Phi(0)\big) + \lambda\psi(p)Y(p), \quad (6)$$

where

$$\psi(p) = L\big(\Phi(\omega)\big) = \int_0^\infty \Phi(\omega)e^{-p\omega}d\omega,$$

$$Y(p) = L\big(f(x)\big) = \int_0^\infty f(x)e^{-px}dx$$

Taking into account the convolution theorem, we get

$$\psi(p)Y(p) = L\left(\int_0^\omega \big(\Phi(\omega - x)f(x)\big)\right)dx$$

Expressing from (6) $\psi(p)$, we obtain

$$\psi(p) = \frac{(-c\Phi(0))}{(\lambda - c\beta - \lambda Y(p))}$$

Applying the inverse Laplace transform, we receive up to a constant factor

$$\Phi(\omega) = -c\Phi(0)\frac{1}{2\pi i}\int_{-i=R_c p}^{i=R_c p} \big(\lambda - cp - \lambda Y(p)\big)e^{ip\omega}d\omega \quad (7)$$

where $the\ (i)^2 = -1$ is an imaginary unit.

We also note that, in practice, cannot restore the original can be restored without using the expression (7).

The practical application of the results. The proposed method will be illustrated for the exponential distribution case, where the distribution density has the following form:

$$f(x) = \begin{cases} \frac{1}{\beta}e^{-\frac{1}{\beta}x}, & x > 0 \\ 0, & x \le 0 \end{cases} \quad (8)$$

In this case, the equation (5) is as follows:

$$\lambda\psi(\omega) = \mathrm{c}\Phi'(\omega) + \lambda\int_0^{\omega} \Phi(\omega - x)\frac{1}{\beta}e^{-\frac{1}{\beta}x}dx$$

Using the above procedure, we obtain

$$\lambda\psi(p) = \mathrm{c}\big(p\psi(p) - \Phi(0)\big) + \frac{\lambda}{\beta}\psi(p)\frac{1}{\gamma + \frac{1}{\beta}},$$

where the

$$\psi(p) = \Phi(0)\left(\frac{\mathrm{c}_1}{\gamma} + \frac{c_2}{\gamma - \left(\frac{\lambda}{c} - \frac{1}{\beta}\right)}\right),$$

$$c_1 = -\frac{\frac{1}{\beta}}{\frac{\lambda}{c} - \frac{1}{\beta}}, \qquad c_2 = -\frac{\frac{\lambda}{c}}{\frac{\lambda}{c} - \frac{1}{\beta}}$$

Applying the method of inverse Laplace transform, we get up to a constant factor

$$\Phi(\omega) = \frac{\Phi(0)}{\frac{\lambda}{\mathrm{c}} - \frac{1}{\beta}}\left(-\frac{1}{\beta} + \frac{\lambda}{\mathrm{c}}e^{\left(\frac{\lambda}{c} - \frac{1}{\beta}\right)x}\right)$$

Given the fact that for the Laplace function $\Phi(\infty) = 1$, we get a formula for the assessment of insurance company`s paying capacity, depending on γ, ω, β:

$$\Phi(\omega) = 1 - \frac{\lambda\beta}{\mathrm{c}}e^{\left(\frac{\lambda}{c} - \frac{1}{\beta}\right)x} = 1 - \frac{1}{1+\gamma}e^{-\frac{\gamma x}{(1+\gamma)m}} \qquad (9)$$

the analysis of the relation (9) implies that $\Phi(\omega)$ depends on the initial capital ω, the load factor γ and the parameter β that is included in the exponential distribution law.

If $m = \beta = 0,78$, $\lambda = 4$ million USD. The Fig. 1 demonstrates the dependence of $\Phi(\omega)$ for $0 \le x \le 20$ (months). As seen in Figure 1, the probability of non-bankruptcy 0.9 at a specified set of parameters is achieved in 20 months of operation of the insurance company. This demonstrates the rationality of optimization of investment options at the initial stage. In particular, one of the methods to maximize the probability of non-bankruptcy is to diversify investments.

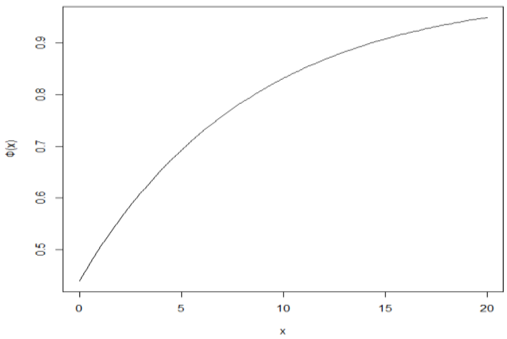

Fig. 1 Dependence of insurance company solvency on distribution parameters

Conclusions. The evaluation of the probability of non-bankruptcy of the insurance company with an initial capital ω is found in the assumption that the number of payments over a time interval $(0, t)$ is

presented as a Poisson distribution law. An analytical condition for the assessment of the company's solvency is achieved by employing the tools of complex analysis. It is shown that in the case of an arbitrary value of payments distribution law to obtain an analytical expression for $\Phi(\omega)$ is possible with the use of a classical inequality of Cramer - Lundberg. A graphical interpretation of the results is presented.

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Received for publication 22.06.2016